Importance Sampling Techniques for Path Tracing in Participating Media

Christopher Kulla Marcos Fajardo

Requirements

- Area Lights
- Global illumination
- Motion Blur of camera, lights, surfaces and volumes
- Procedural Shaders
- Minimal memory usage
- Progressive rendering
- Artist friendly quality controls

Volume Rendering Equation [Kajiya and Von Herzen, 1984]

$$L(\mathbf{x}, \vec{\mathbf{w}}) = \int_a^b e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(L_e\left(\mathbf{x}_t, \vec{\mathbf{w}}\right) + \sigma_s\left(\mathbf{x}_t\right) \int_{S^2} \rho(\vec{\mathbf{w}}, \vec{\mathbf{v}}) L(\mathbf{x}_t, \vec{\mathbf{v}})\,\mathrm{d}\vec{\mathbf{v}} \right) \mathrm{d}t$$

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- *Nested* integral for incoming light

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- *Nested* integral for incoming light
- *Nested* integral for heterogeneous media

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- *Nested* integral for incoming light
- *Nested* integral for heterogeneous media
- *Recursive* integral for multiple scattering

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- *Nested* integral for incoming light
- *Nested* integral for heterogeneous media
- *Recursive* integral for multiple scattering

Our approach

New importance sampling techniques for each problem

Importance Sampling Incoming Light

$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \int_a^b e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(L_e\left(\mathbf{x}_t, \overrightarrow{\mathbf{w}}\right) + \sigma_s\left(\mathbf{x}_t\right) \int_{S^2} \rho(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{v}}) L(\mathbf{x}_t, \overrightarrow{\mathbf{v}})\,\mathrm{d}\overrightarrow{\mathbf{v}} \right) \mathrm{d}t$$

Homogenous media ▼

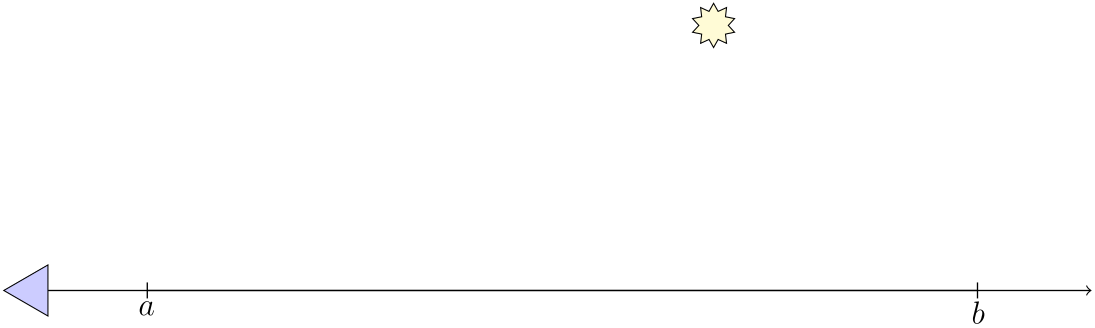
$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \int_a^b e^{-\sigma_t t} \left(\sigma_s \int_{S^2} \rho(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{v}}) L(\mathbf{x}_t, \overrightarrow{\mathbf{v}})\,\mathrm{d}\overrightarrow{\mathbf{v}} \right) \mathrm{d}t$$

Point light ▼

$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \sigma_s \int_a^b e^{-\sigma_t t} \rho(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{x}_t\mathbf{c}}) L(\mathbf{x}_t, \overrightarrow{\mathbf{x}_t\mathbf{c}})\,\mathrm{d}t$$

Single Scattering Equation for Point Light

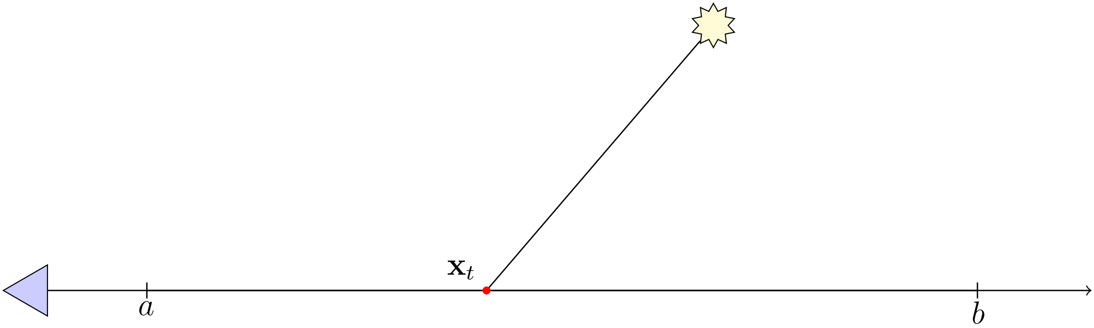
Integrate point light contribution along the ray



$$L(\mathbf{x}, \vec{\mathbf{w}}) = \sigma_s \int_a^b \qquad \mathrm{d}t$$

Single Scattering Equation for Point Light

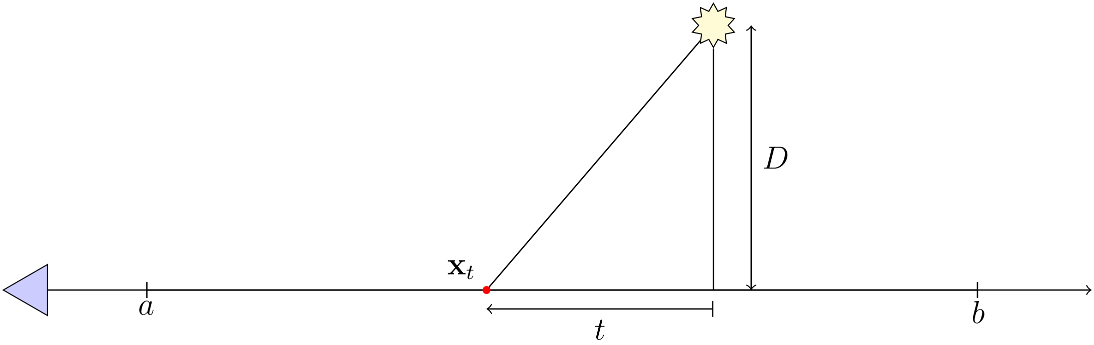
Radiance reaching the ray varies as $1/r^2$



$$L(\mathbf{x}, \vec{\mathbf{w}}) = \sigma_s \int_a^b \qquad \frac{\Phi}{\|\mathbf{x}_t - \mathbf{c}\|^2} \, \mathrm{d}t$$

Single Scattering Equation for Point Light

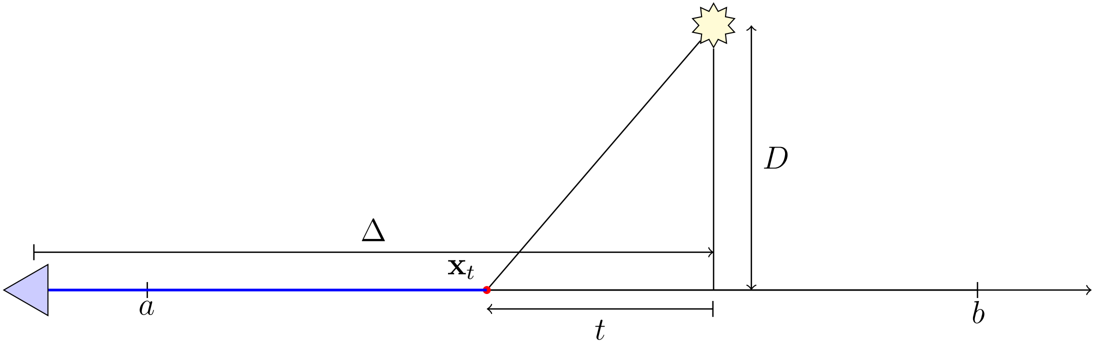
Express in terms of t



$$L(\mathbf{x}, \vec{\mathbf{w}}) = \sigma_s \int_a^b \qquad \frac{\Phi}{D^2+t^2}\,\mathrm{d}t$$

Single Scattering Equation for Point Light

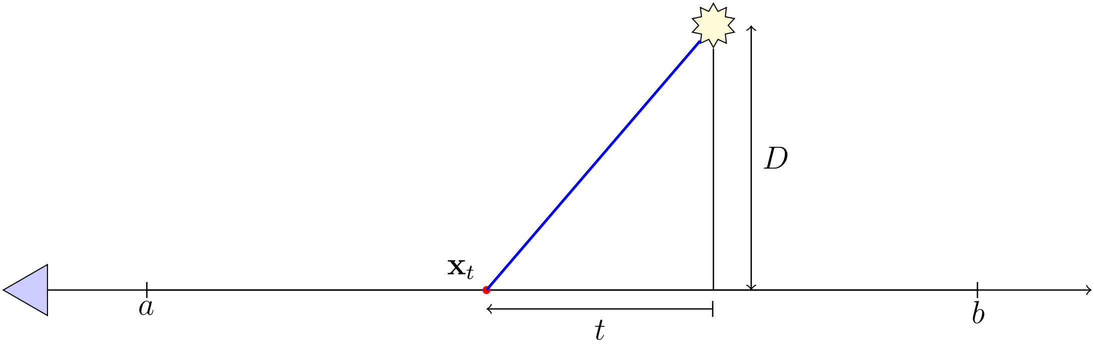
Account for extinction up to sample point



$$L(\mathbf{x}, \vec{\mathbf{w}}) = \sigma_s \int_a^b e^{-\sigma_t(t+\Delta)} \frac{\Phi}{D^2+t^2}\,\mathrm{d}t$$

Single Scattering Equation for Point Light

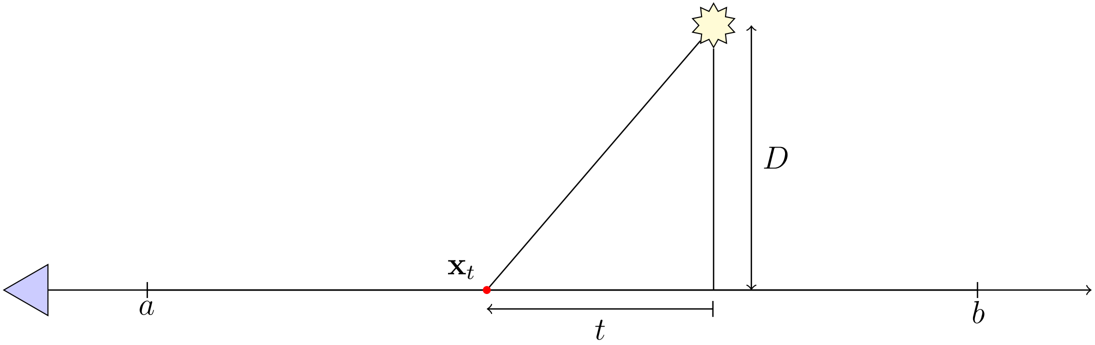
Add extinction towards the light



$$L(\mathbf{x}, \vec{\mathbf{w}}) = \sigma_s \int_a^b e^{-\sigma_t (t+\Delta)} e^{-\sigma_t \sqrt{D^2+t^2}} \qquad \frac{\Phi}{D^2+t^2} \, \mathrm{d}t$$

Single Scattering Equation for Point Light

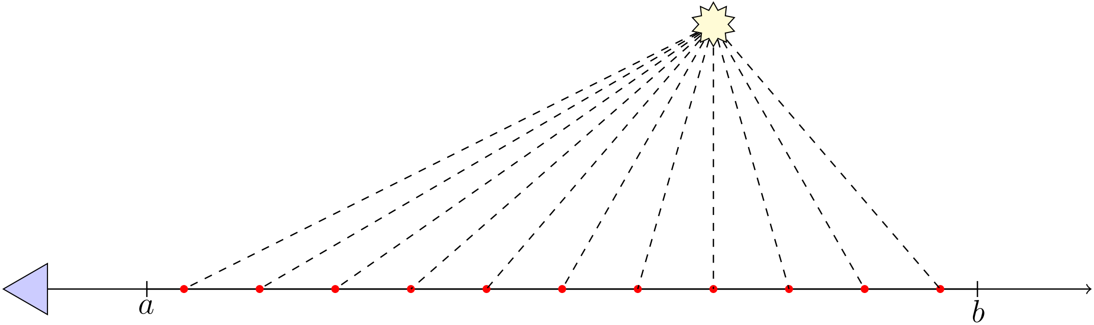
Finally add phase function and visibility



$$L(\mathbf{x}, \vec{\mathbf{w}}) = \sigma_s \int_a^b e^{-\sigma_t (t+\Delta)} e^{-\sigma_t \sqrt{D^2+t^2}} \rho(\vec{\mathbf{w}}, \overrightarrow{\mathbf{x}_t \mathbf{c}}) V(\mathbf{x}_t, \mathbf{c}) \frac{\Phi}{D^2+t^2} \, \mathrm{d}t$$

Single Scattering Equation for Point Light

What is the best sample distribution for Monte Carlo integration?



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Importance sampling for point lights

$$L(\mathbf{x}, \vec{\mathbf{w}}) = \sigma_s \int_a^b e^{-\sigma_t (t+\Delta)} e^{-\sigma_t \sqrt{D^2+t^2}} \rho\left(\vec{\mathbf{w}}, \vec{\mathbf{v}}_t\right) V\left(\mathbf{c}, \mathbf{x}_t\right) \frac{\Phi}{D^2+t^2} \, \mathrm{d}t$$

Only two terms can be easily integrated and inverted

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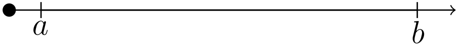
Transmission $e^{-\sigma_t t}$



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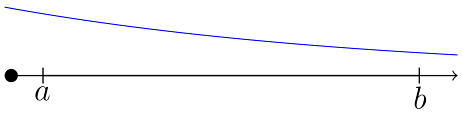

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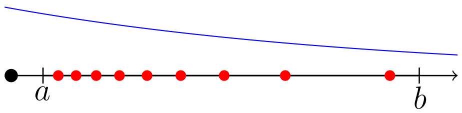

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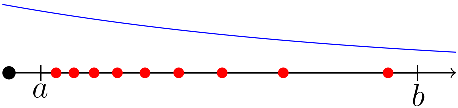
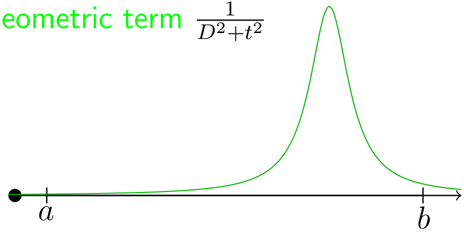


Geometric term $\frac{1}{D^2+t^2}$



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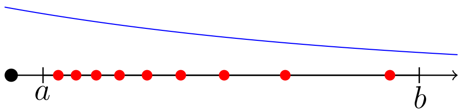

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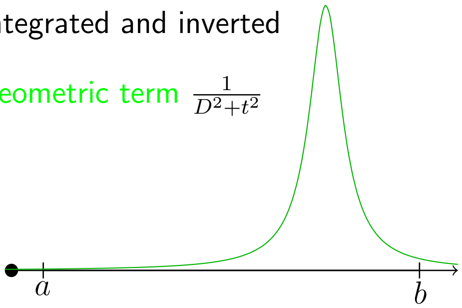

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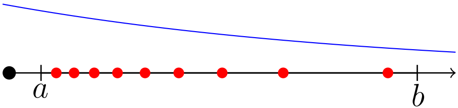

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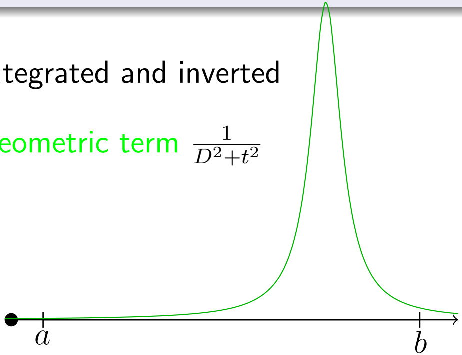

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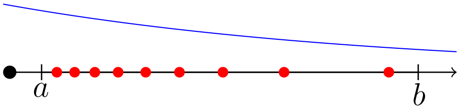

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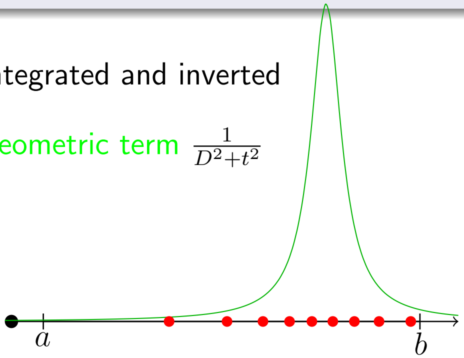
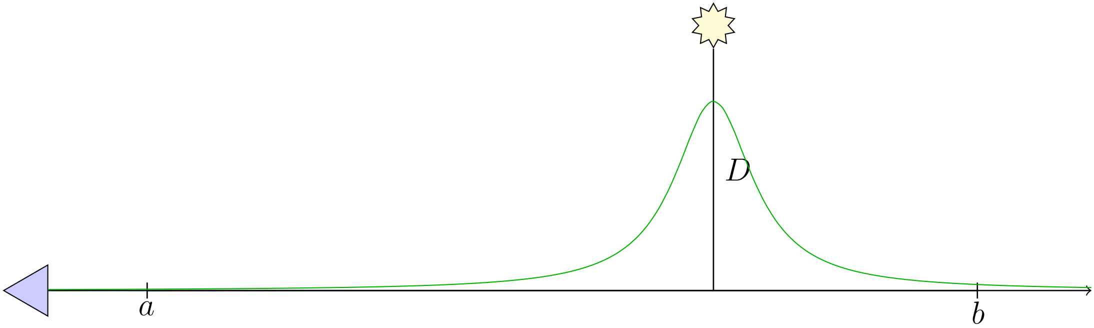


Importance sampling for point lights

Goal is to get a pdf proportional to geometric term:

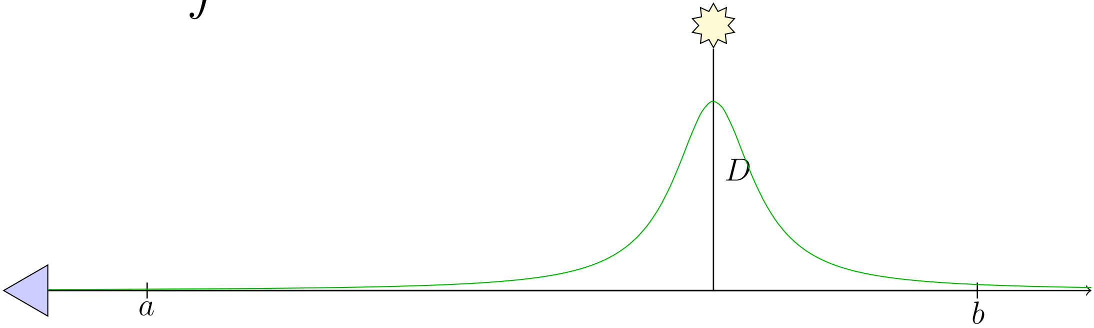
$$\mathsf{pdf}(t) \propto \frac{1}{D^2+t^2}$$



Importance sampling for point lights

Integrate pdf to obtain cdf:

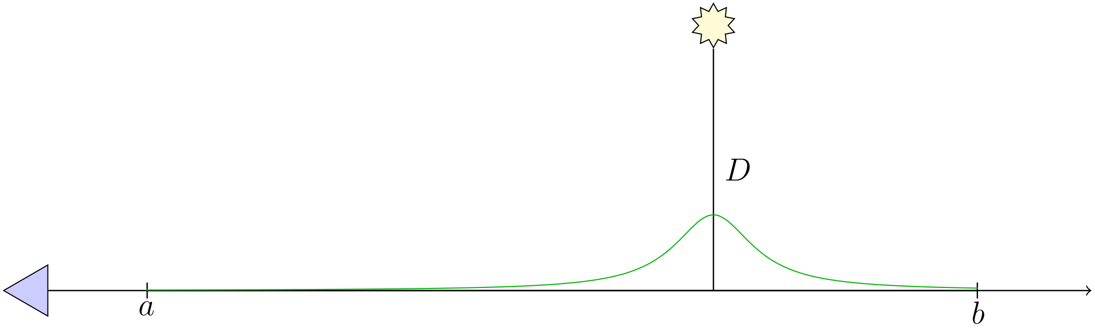
$$\mathsf{cdf}(t) = \int \frac{1}{D^2+t^2}\,dt = \frac{1}{D}\tan^{-1}\frac{t}{D}$$



Importance sampling for point lights

Use cdf to normalize over $[a, b]$:

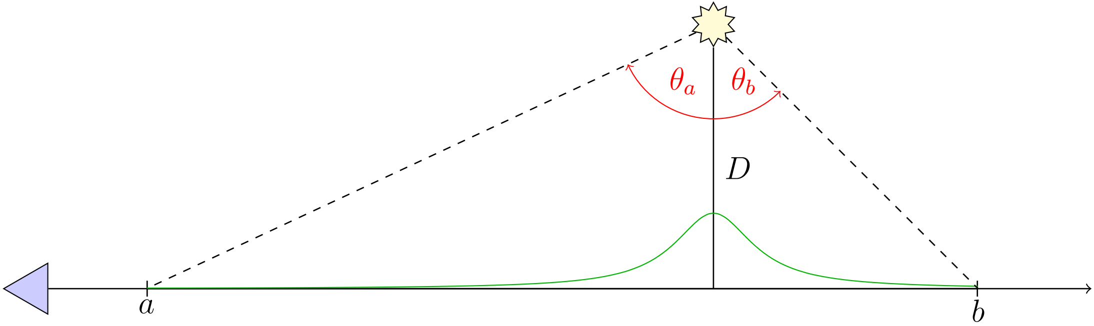
$$\text{pdf}(t) = \frac{D}{(\tan^{-1}\frac{b}{D} - \tan^{-1}\frac{a}{D})(D^2 + t^2)}$$



Importance sampling for point lights

Use cdf to normalize over $[a, b]$:

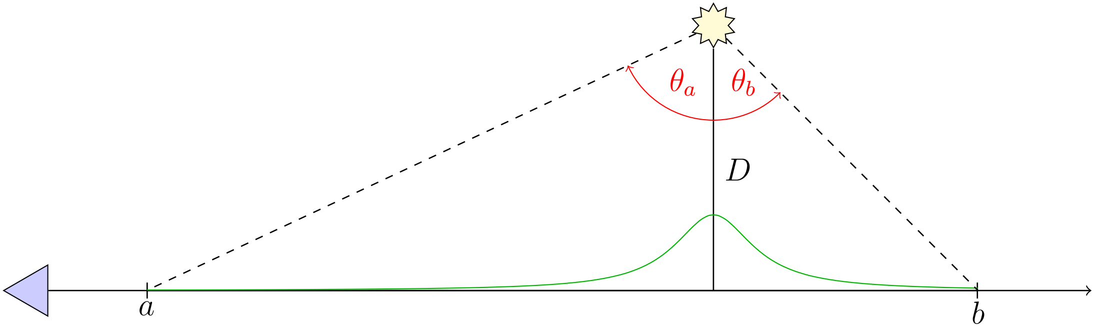
$$\mathsf{pdf}(t) = \frac{D}{(\theta_b - \theta_a)(D^2 + t^2)}$$



Importance sampling for point lights

Invert cdf to obtain distribution for $\xi_i \in [0, 1)$:

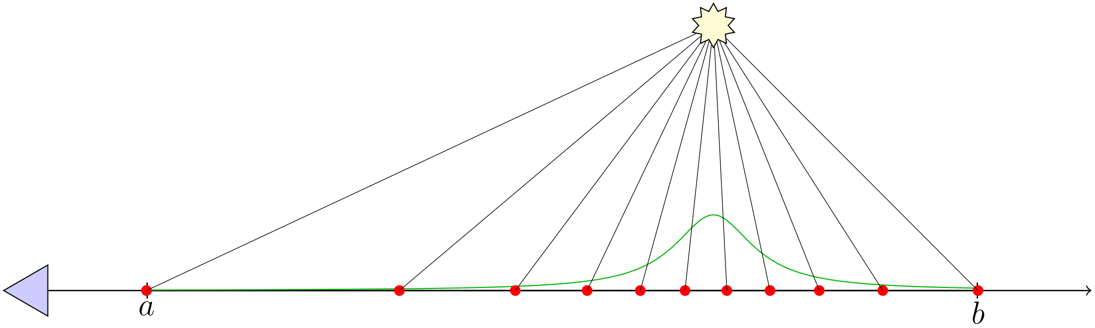
$$t_i = D \tan\left(\left(1 - \xi_i\right)\theta_a + \xi_i \theta_b\right)$$



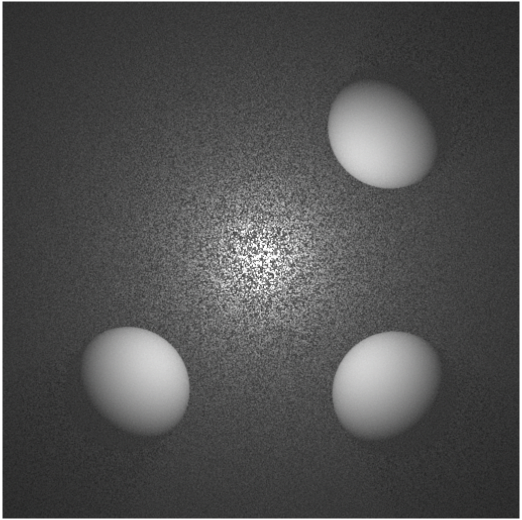
Importance sampling for point lights

Sample distribution is *equi-angular*

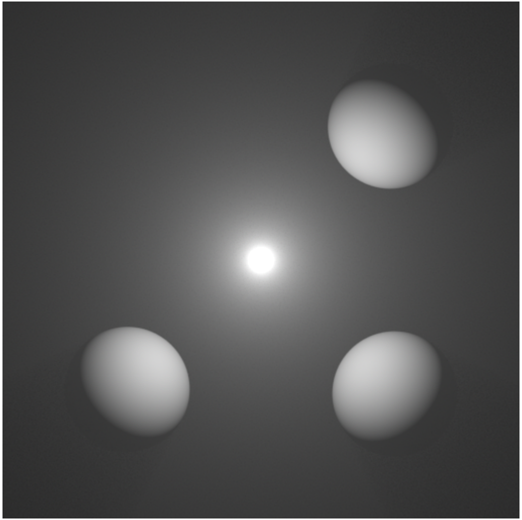
$$t_i = D \tan\left(\left(1 - \xi_i\right)\theta_a + \xi_i \theta_b\right)$$



Results with 16 samples/pixel

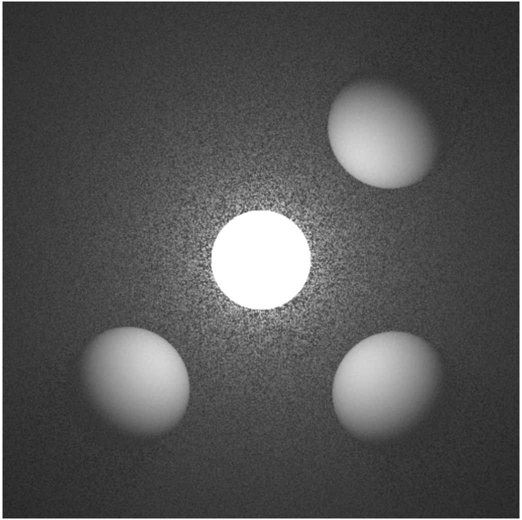


Density sampling

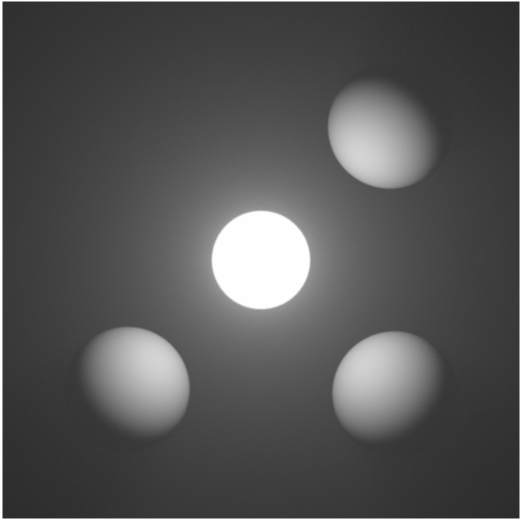


Our method

Sphere lights can use same equations! ($\Omega \propto 1/r^2$)



Density sampling



Our method

Importance sampling for arbitrary area lights

$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \sigma_s \int_a^b e^{-\sigma_t t} e^{-\sigma_t \|\mathbf{x}_t - \mathbf{c}\|} \rho\left(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{v}_t}\right) V\left(\mathbf{c}, \mathbf{x}_t\right) \frac{\Phi}{\|\mathbf{x}_t - \mathbf{c}\|^2} \, \mathrm{d}t$$

Area light ▼

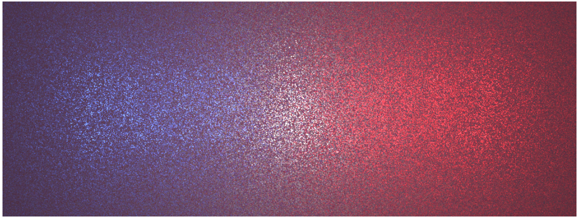
$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \sigma_s \int_a^b e^{-\sigma_t t} \int_A \rho(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{x}_t \mathbf{y}}) e^{-\sigma_t \|\mathbf{x}_t - \mathbf{y}\|} \frac{\cos \theta_{\mathbf{y}}}{\|\mathbf{x}_t - \mathbf{y}\|^2} V(\mathbf{x}_t, \mathbf{y}) L_e(\mathbf{y}, \overrightarrow{\mathbf{y}\mathbf{x}_t}) \, \mathrm{d}A\left(\mathbf{y}\right) \, \mathrm{d}t$$

Importance sampling for arbitrary area lights

- Same two choices for importance sampling

Importance sampling for arbitrary area lights

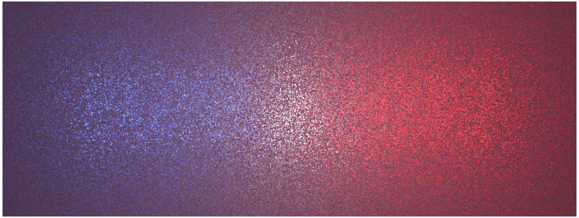
- Same two choices for importance sampling
  - Transmission $e^{-\sigma_t t}$



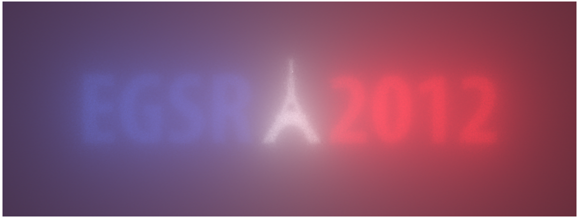
Density sampling

Importance sampling for arbitrary area lights

- Same two choices for importance sampling
  - Transmission $e^{-\sigma_t t}$
  - Geometric term $1/\|\mathbf{x}_t - \mathbf{y}\|^2$



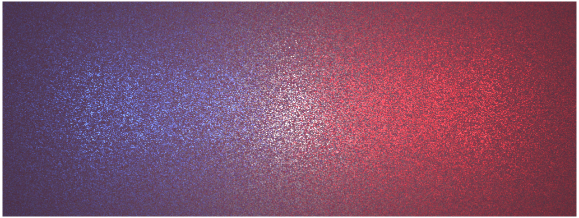
Density sampling



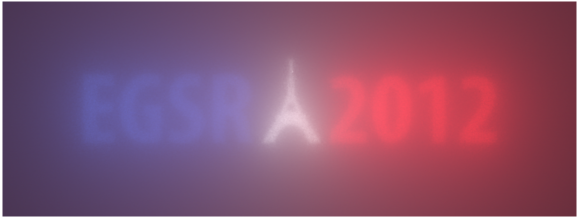
Equi-angular sampling

Importance sampling for arbitrary area lights

- Same two choices for importance sampling
  - Transmission $e^{-\sigma_t t}$
  - Geometric term $1/\|\mathbf{x}_t - \mathbf{y}\|^2$
- To sample geometric term, choose $\mathbf{y}$ before $\mathbf{x}_t$



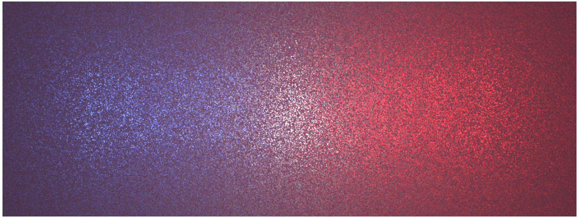
Density sampling



Equi-angular sampling

Importance sampling for arbitrary area lights

- Same two choices for importance sampling
  - Transmission $e^{-\sigma_t t}$
  - Geometric term $1/\|\mathbf{x}_t - \mathbf{y}\|^2$
- To sample geometric term, choose $\mathbf{y}$ before $\mathbf{x}_t$
- Can combine both line sampling strategies by MIS
- More details in paper about using the phase function



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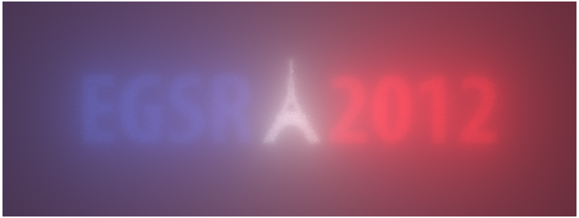


Equi-angular sampling

Importance Sampling for Heterogeneous Media

Importance sampling in heterogeneous media

$$L(\mathbf{x}, \vec{\mathbf{w}}) = \int_a^b e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(L_e\left(\mathbf{x}_t, \vec{\mathbf{w}}\right) + \sigma_s\left(\mathbf{x}_t\right) \int_{S^2} \rho(\vec{\mathbf{w}}, \vec{\mathbf{v}}) L(\mathbf{x}_t, \vec{\mathbf{v}})\,\mathrm{d}\vec{\mathbf{v}} \right) \mathrm{d}t$$

- Transmission term contains an integral

Importance sampling in heterogeneous media

$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \int_a^b e^{-\int_0^{t} \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(L_e\left(\mathbf{x}_t, \overrightarrow{\mathbf{w}}\right) + \sigma_s\left(\mathbf{x}_t\right) \int_{S^2} \rho(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{v}}) L(\mathbf{x}_t, \overrightarrow{\mathbf{v}})\,\mathrm{d}\overrightarrow{\mathbf{v}} \right) \mathrm{d}t$$

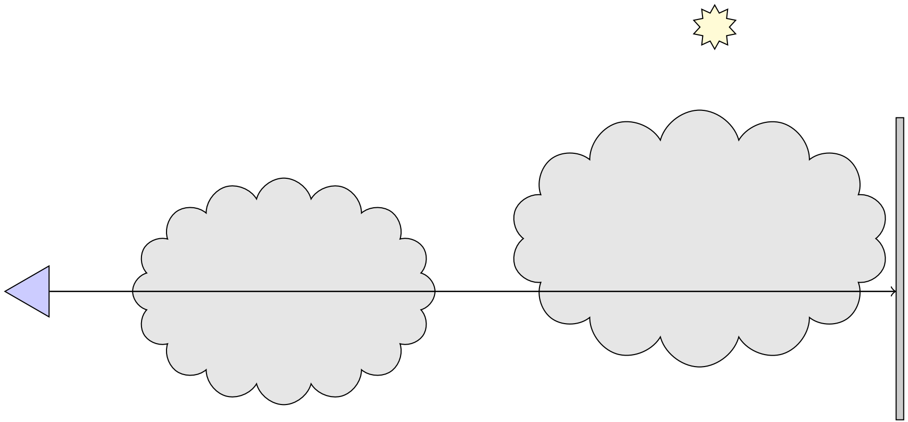
- Transmission term contains an integral
- Integral limit is also integration variable

Importance sampling in heterogeneous media

$$L(\mathbf{x}, \vec{\mathbf{w}}) = \int_a^b e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(L_e\left(\mathbf{x}_t, \vec{\mathbf{w}}\right) + \sigma_s\left(\mathbf{x}_t\right) \int_{S^2} \rho(\vec{\mathbf{w}}, \vec{\mathbf{v}}) L(\mathbf{x}_t, \vec{\mathbf{v}})\,\mathrm{d}\vec{\mathbf{v}} \right) \mathrm{d}t$$

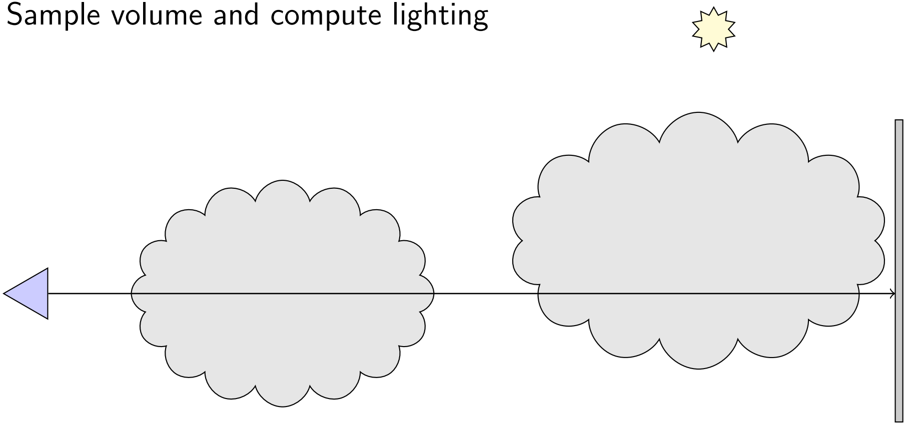
- Transmission term contains an integral
- Integral limit is also integration variable
- Previous method assumed we could take samples at any t

Brute Force Ray Marching [Perlin 1989]



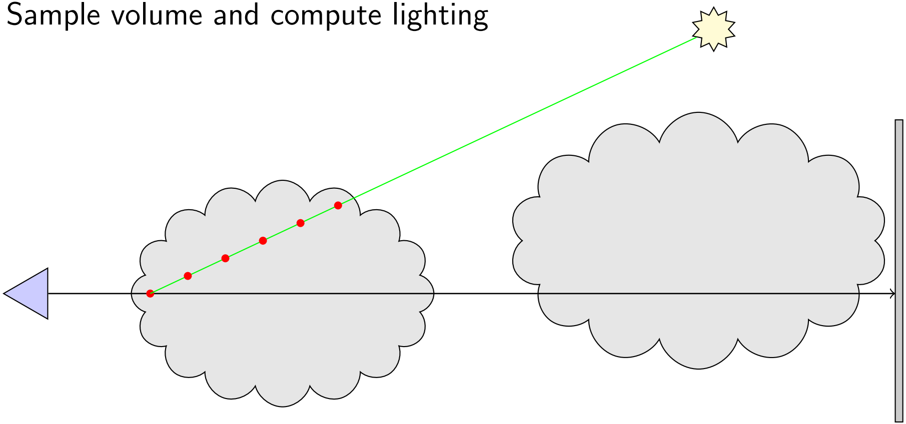
Brute Force Ray Marching [Perlin 1989]

Sample volume and compute lighting



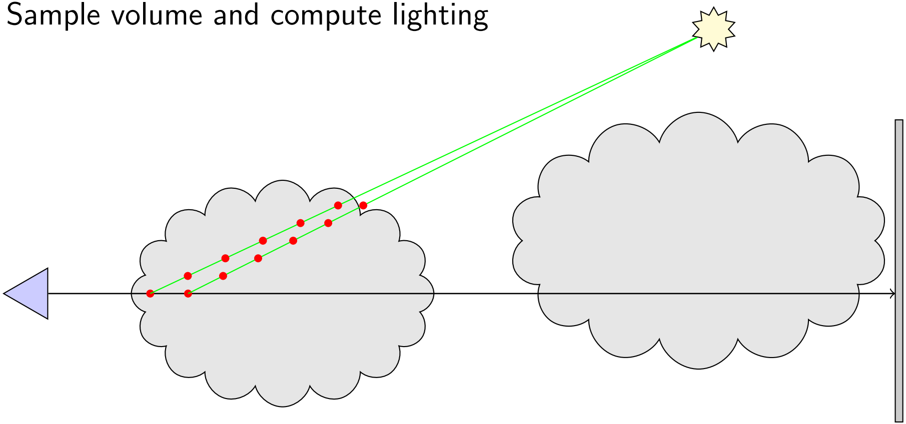
Brute Force Ray Marching [Perlin 1989]

Sample volume and compute lighting



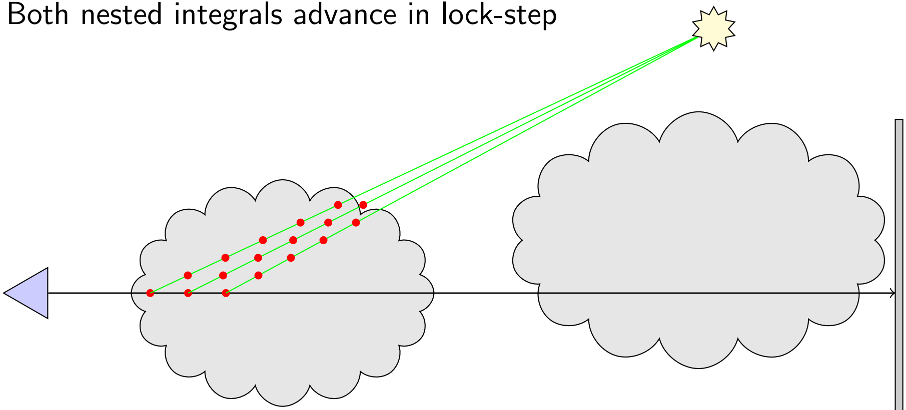
Brute Force Ray Marching [Perlin 1989]

Sample volume and compute lighting



Brute Force Ray Marching [Perlin 1989]

Both nested integrals advance in lock-step



Brute Force Ray Marching [Perlin 1989]

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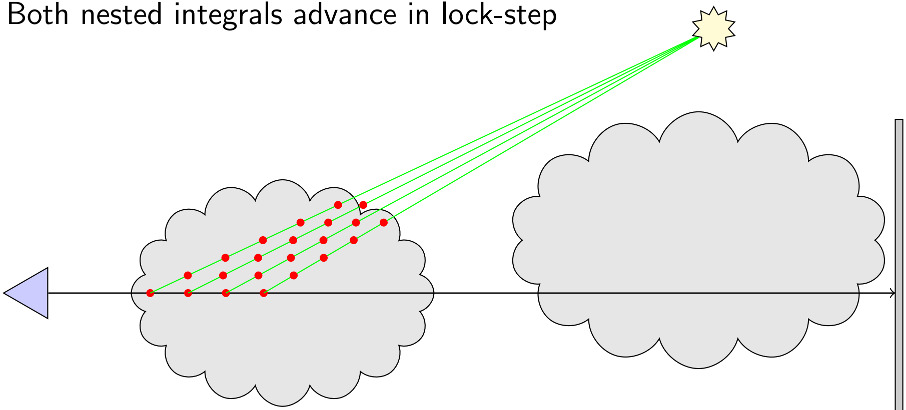

Brute Force Ray Marching [Perlin 1989]

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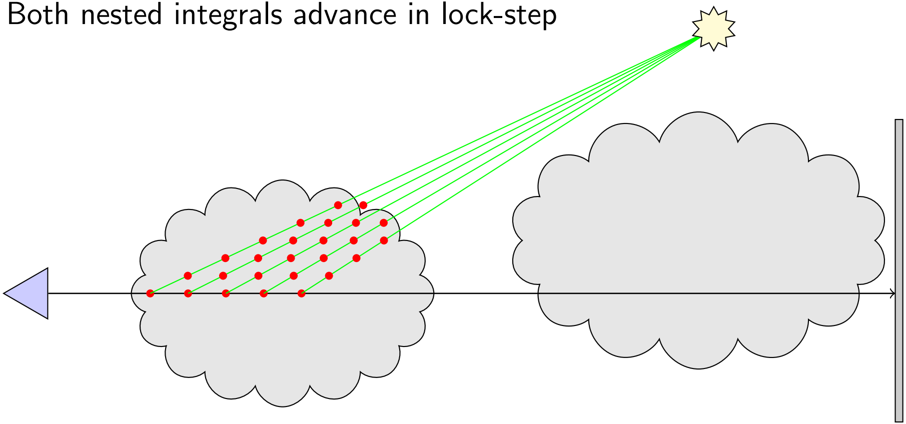
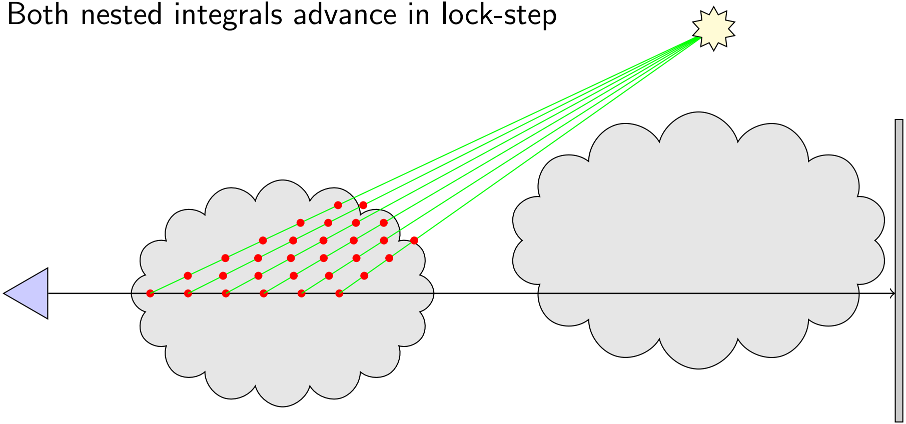


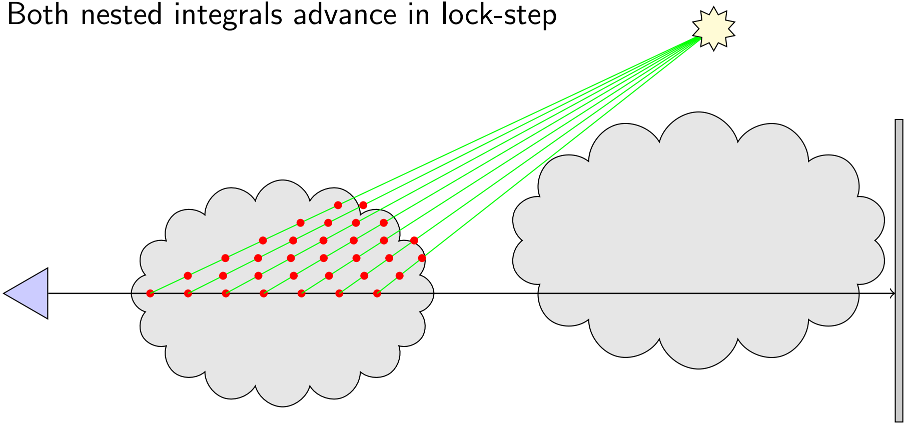
Brute Force Ray Marching [Perlin 1989]

Both nested integrals advance in lock-step



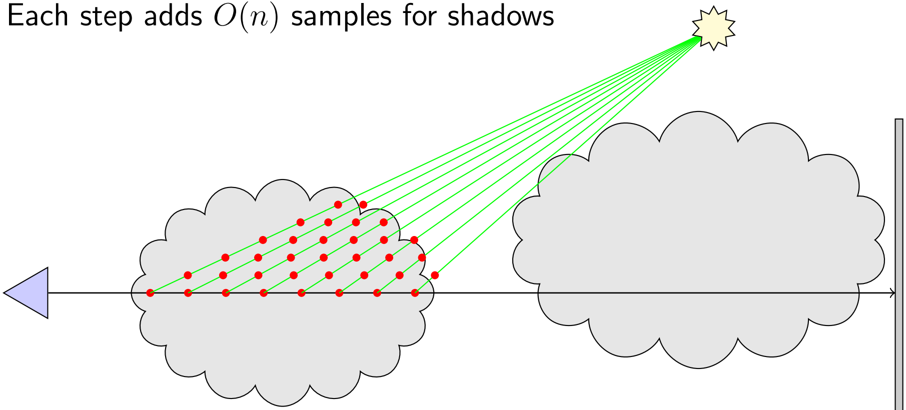
Brute Force Ray Marching [Perlin 1989]

Both nested integrals advance in lock-step



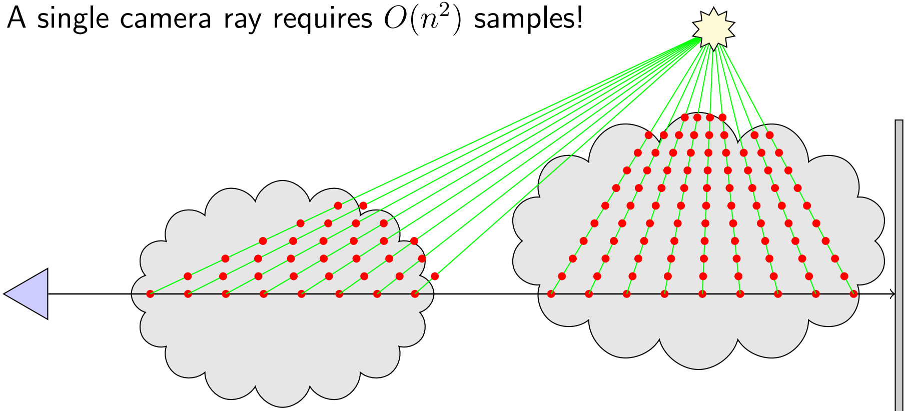
Brute Force Ray Marching [Perlin 1989]

Each step adds $O(n)$ samples for shadows



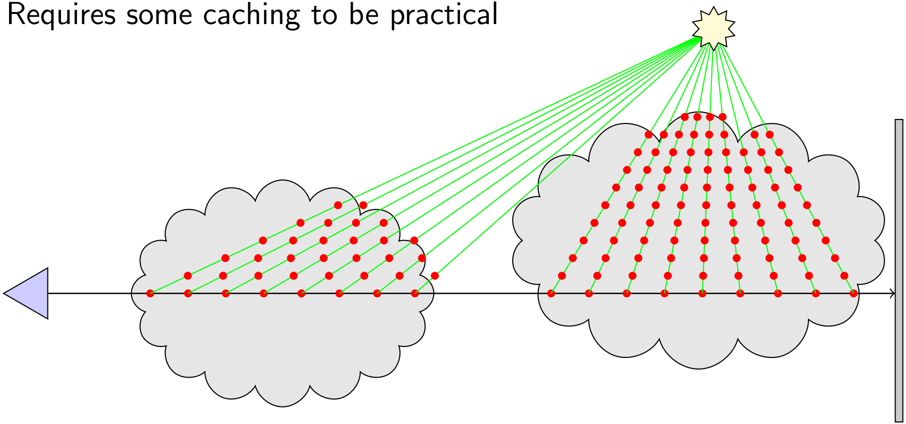
Brute Force Ray Marching [Perlin 1989]

A single camera ray requires $O(n^2)$ samples!

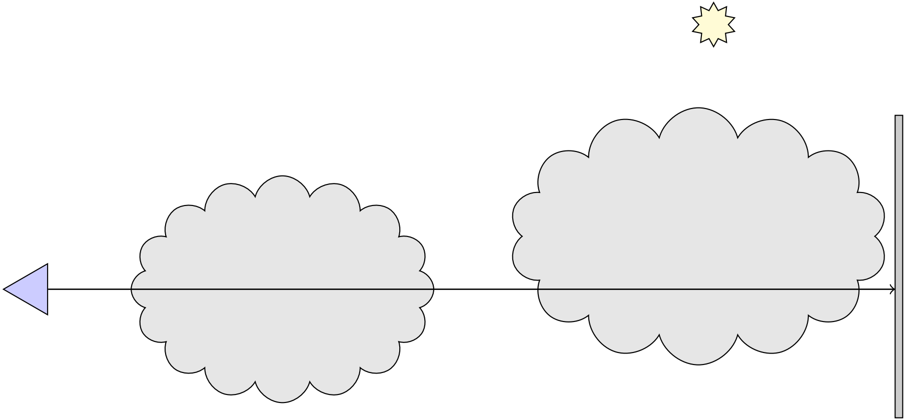


Brute Force Ray Marching [Perlin 1989]

Requires some caching to be practical

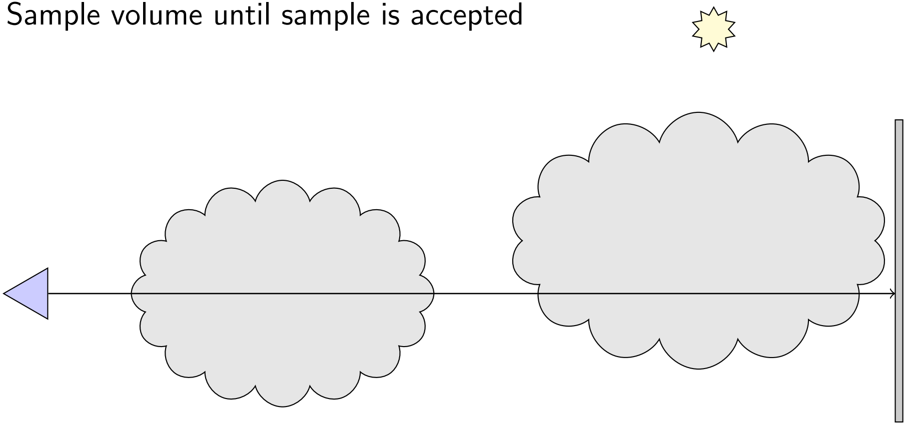


Stochastic Ray Marching [Lafortune et al, 1996]



Stochastic Ray Marching [Lafortune et al, 1996]

Sample volume until sample is accepted



Stochastic Ray Marching [Lafortune et al, 1996]

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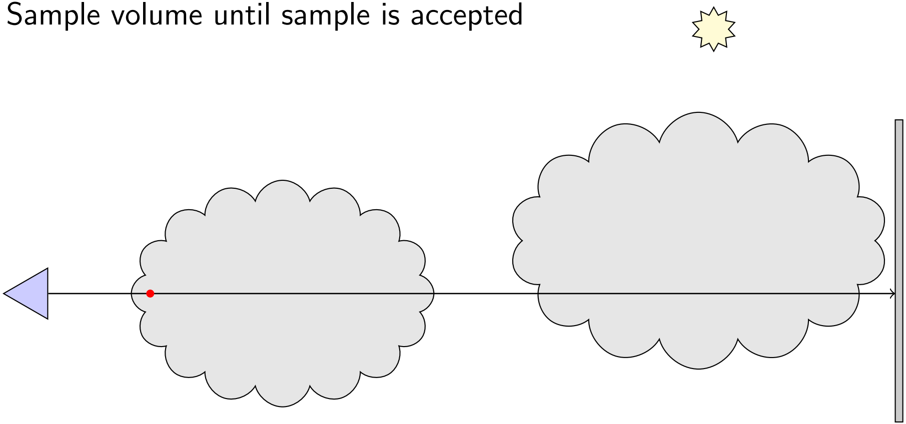

Stochastic Ray Marching [Lafortune et al, 1996]

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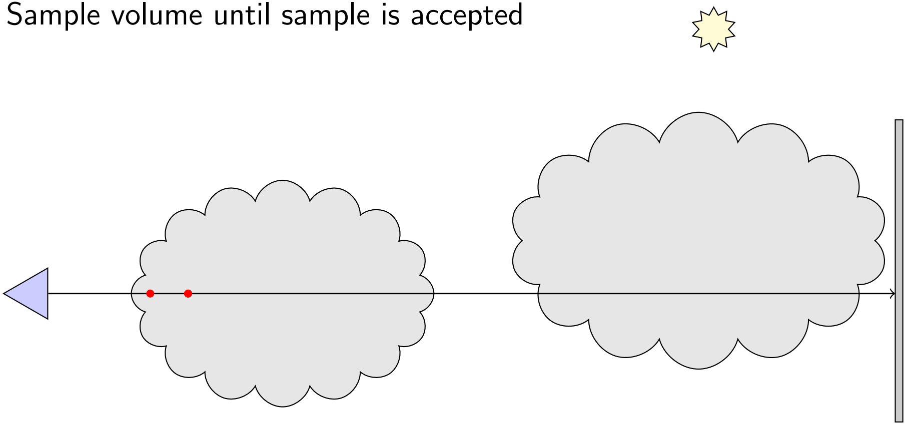
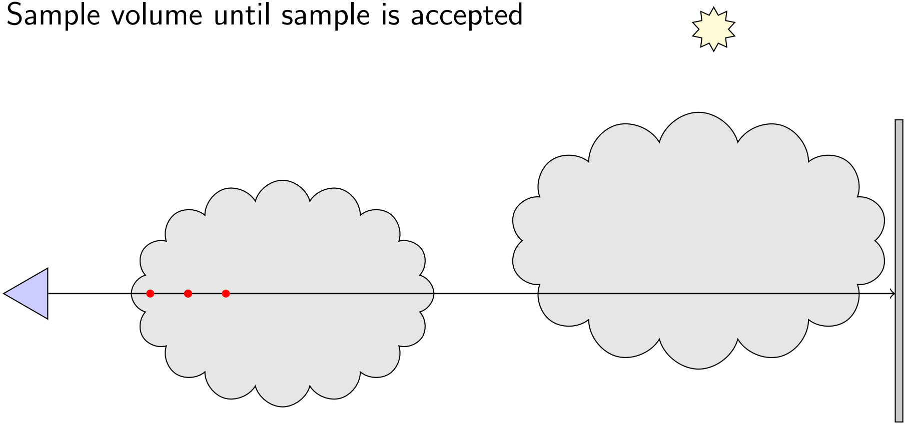


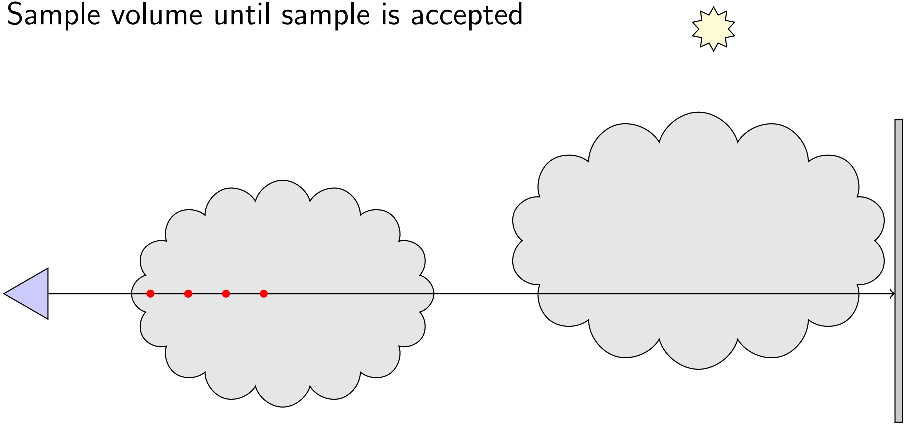
Stochastic Ray Marching [Lafortune et al, 1996]

Sample volume until sample is accepted



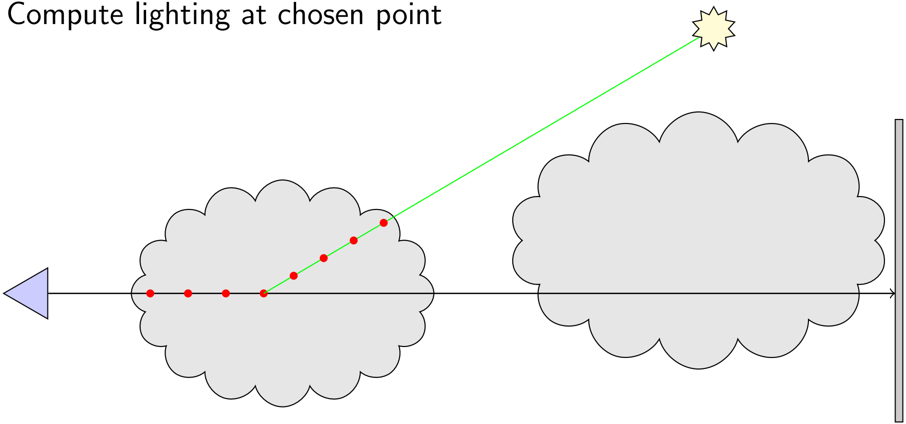
Stochastic Ray Marching [Lafortune et al, 1996]

Sample volume until sample is accepted



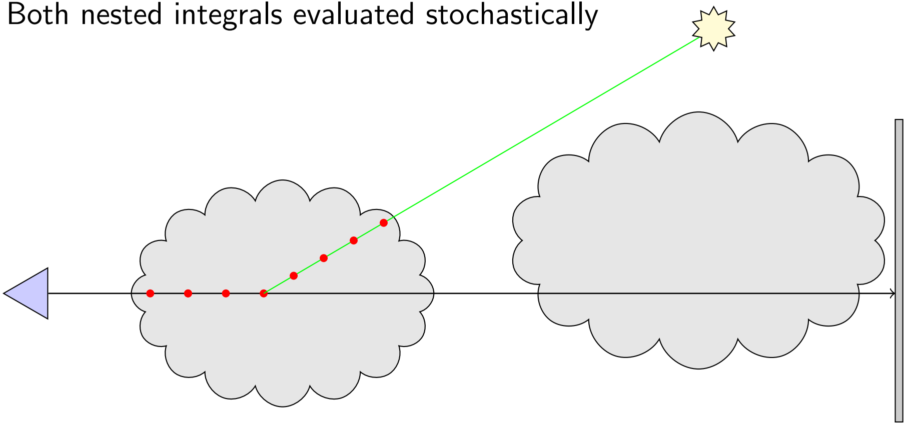
Stochastic Ray Marching [Lafortune et al, 1996]

Compute lighting at chosen point



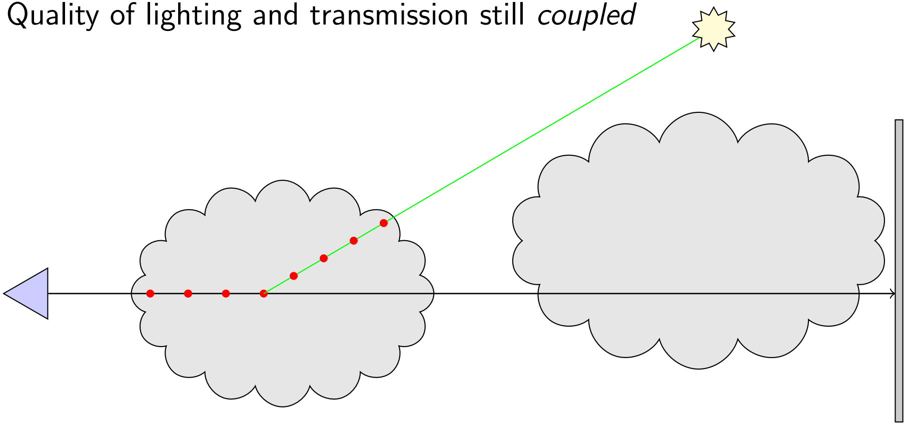
Stochastic Ray Marching [Lafortune et al, 1996]

Both nested integrals evaluated stochastically

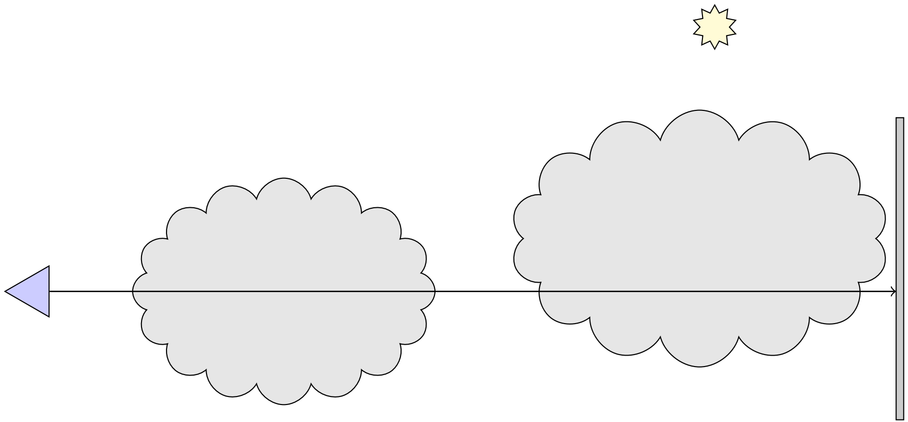


Stochastic Ray Marching [Lafortune et al, 1996]

Quality of lighting and transmission still *coupled*

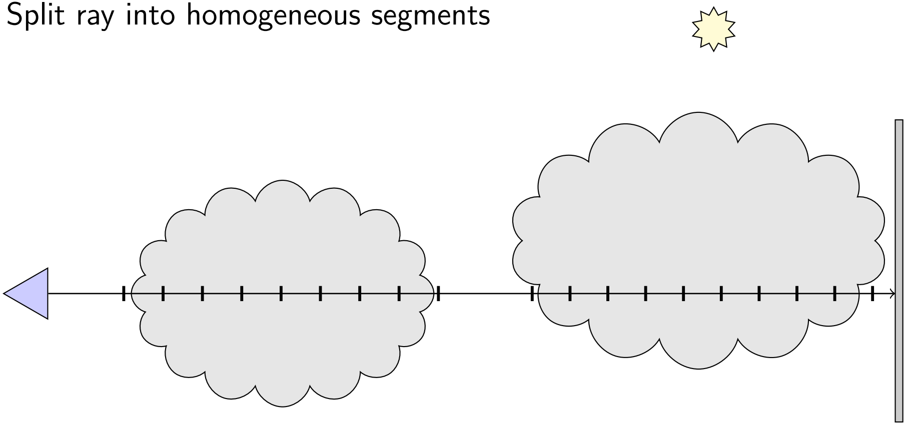


Our approach - Decoupled Ray Marching



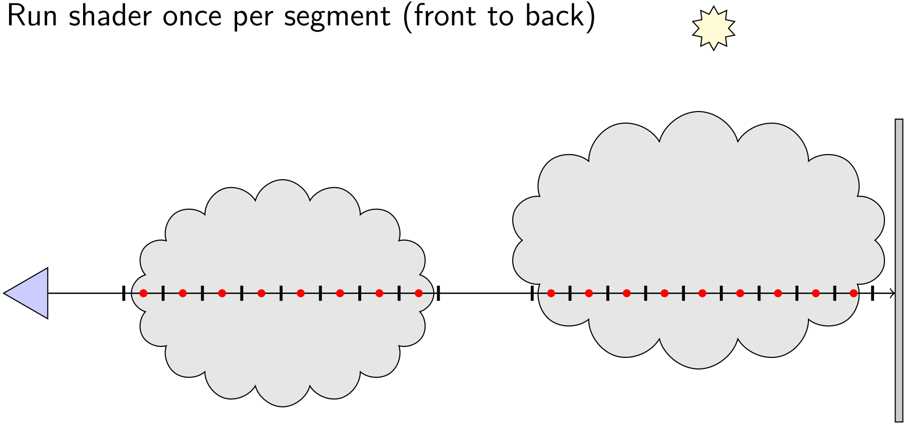
Our approach - Decoupled Ray Marching

Split ray into homogeneous segments



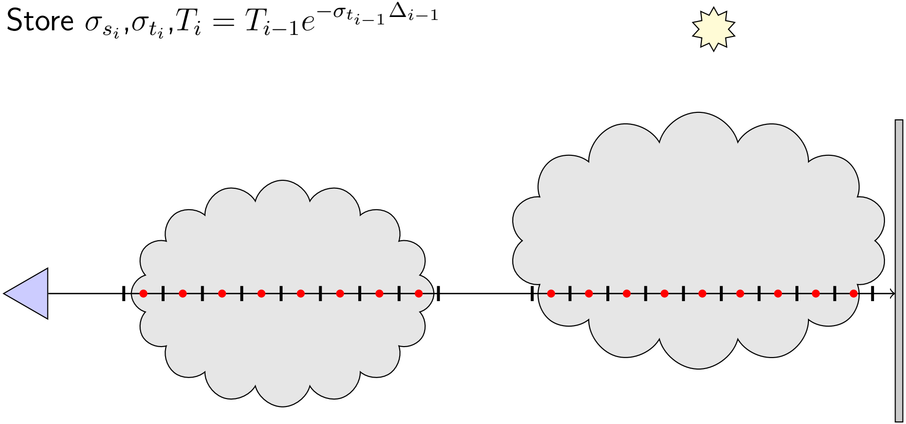
Our approach - Decoupled Ray Marching

Run shader once per segment (front to back)



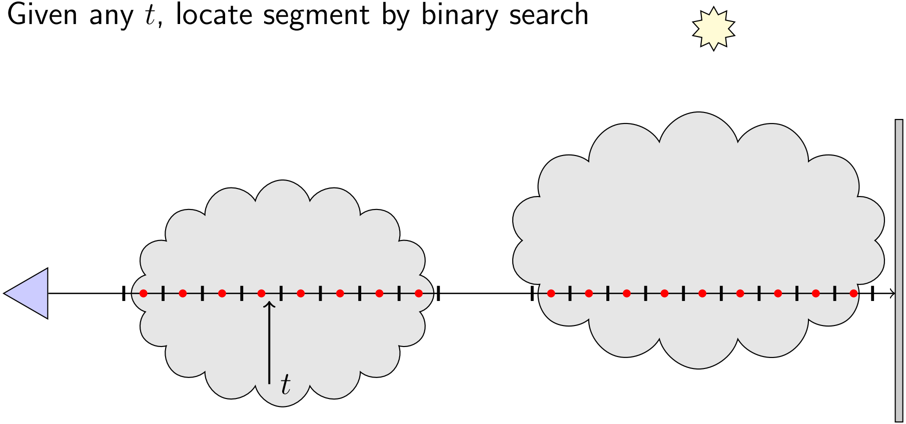
Our approach - Decoupled Ray Marching

Store σ_{s_i},σ_{t_i},$T_i = T_{i-1}e^{-\sigma_{t_{i-1}}\Delta_{i-1}}$



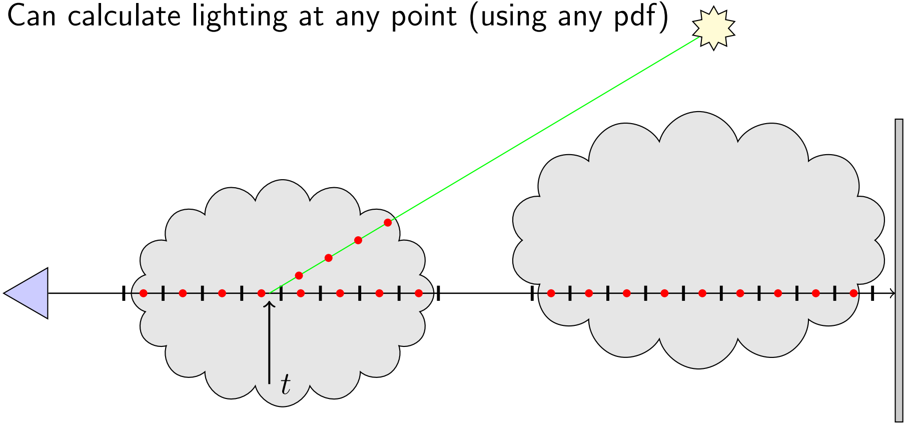
Our approach - Decoupled Ray Marching

Given any t, locate segment by binary search



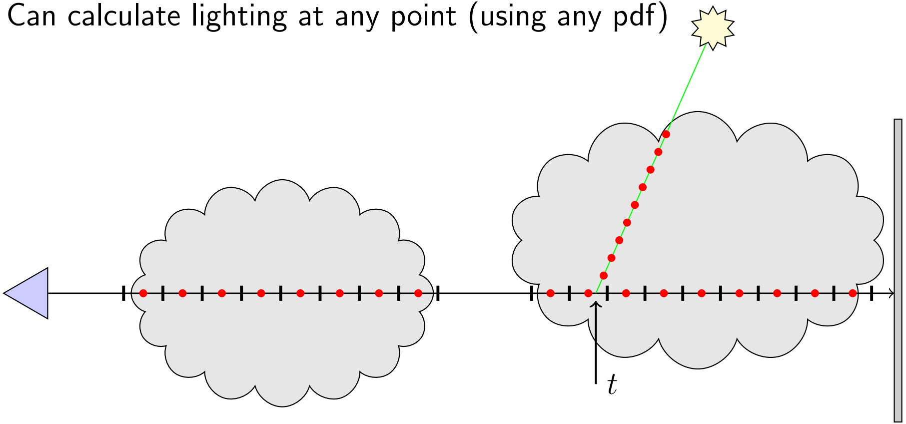
Our approach - Decoupled Ray Marching

Can calculate lighting at any point (using any pdf)

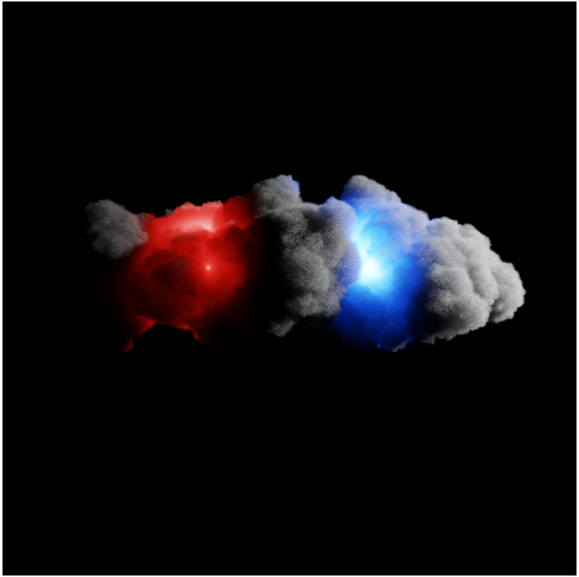


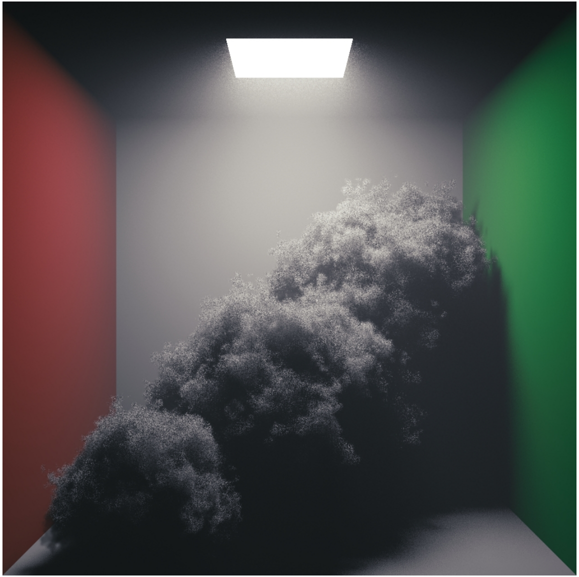
Our approach - Decoupled Ray Marching

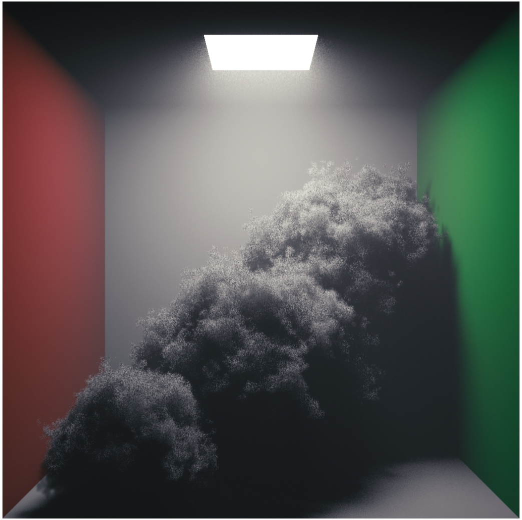
Can calculate lighting at any point (using any pdf)



Equi-angular sampling in heterogeneous media

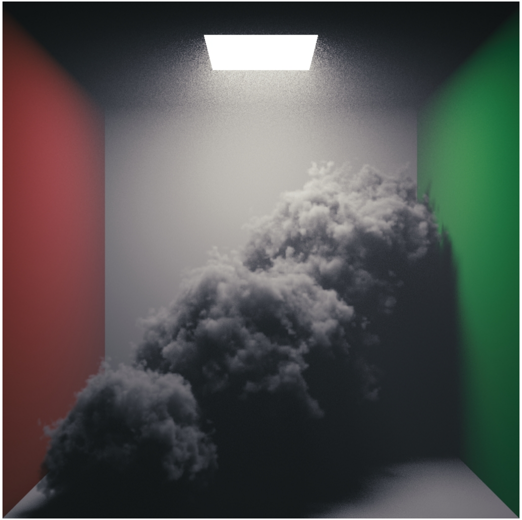






Equi-angular Sampling ($\mathrm{pdf}(t) \propto 1/r^2$)

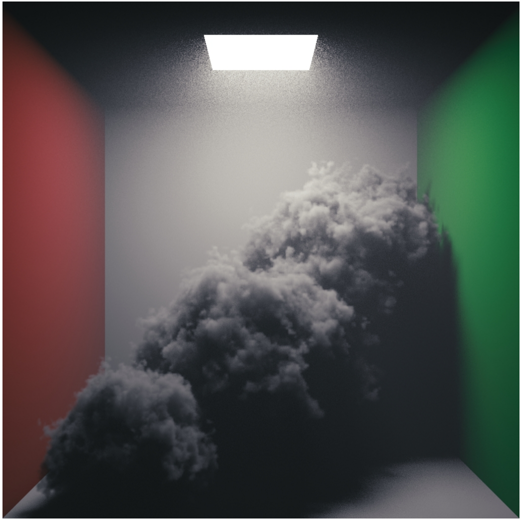
- Equi-angular sampling not optimal everywhere



Density Sampling ($\mathrm{pdf}(t) \propto \sigma_{s_i} T_i$)

$$L(\mathbf{x}, \vec{\mathbf{w}}) = \int_a^b \sigma_s(\mathbf{x}_t)\, e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(\int_{S^2} \rho(\vec{\mathbf{w}}, \vec{\mathbf{v}}) L(\mathbf{x}_t, \vec{\mathbf{v}})\,\mathrm{d}\vec{\mathbf{v}} \right) \mathrm{d}t$$

- Equi-angular sampling not optimal everywhere
- Build discrete pdf from ray marched samples



Density Sampling ($\mathrm{pdf}(t) \propto \sigma_{s_i} T_i$)

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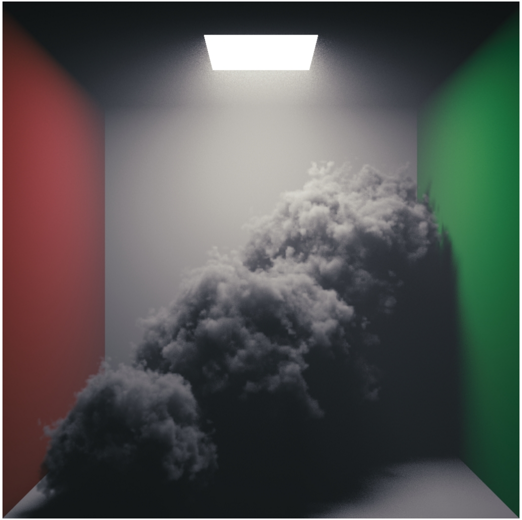
- Equi-angular sampling not optimal everywhere
- Build discrete pdf from ray marched samples
- Samples are focused where $\sigma_s > 0$

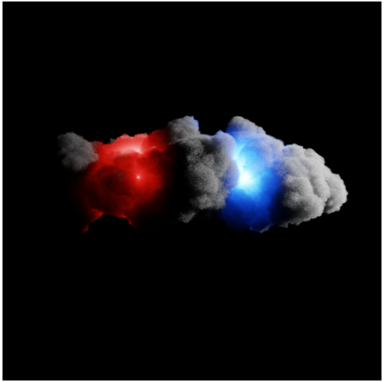


MIS (best of both)

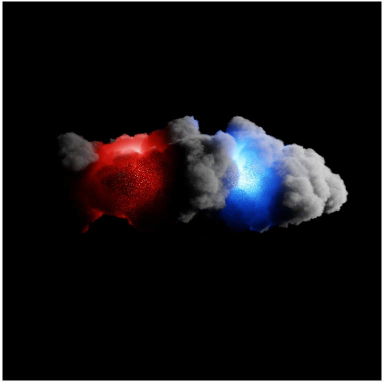
$$L(\mathbf{x}, \vec{\mathbf{w}}) = \int_a^b \sigma_s(\mathbf{x}_t)\, e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(\int_{S^2} \rho(\vec{\mathbf{w}}, \vec{\mathbf{v}}) L(\mathbf{x}_t, \vec{\mathbf{v}})\,\mathrm{d}\vec{\mathbf{v}} \right) \mathrm{d}t$$

- Equi-angular sampling not optimal everywhere
- Build discrete pdf from ray marched samples
- Samples are focused where $\sigma_s > 0$
- Can combine multiple pdfs by MIS

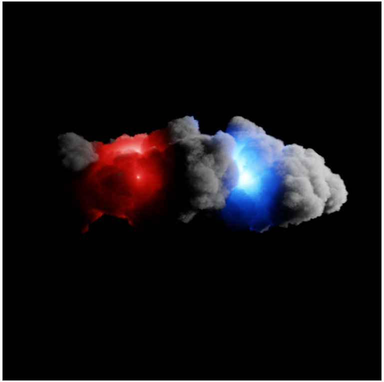
Results - Decoupled Ray Marching



Equi-angular Sampling
$(\mathrm{pdf}(t) \propto 1/r^2)$



Density Sampling
$(\mathrm{pdf}(t) \propto \sigma_{s_i} T_i)$



MIS
(best of both)

Multiple Scattering

$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \int_a^b e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \left(L_e\left(\mathbf{x}_t, \overrightarrow{\mathbf{w}}\right) + \sigma_s\left(\mathbf{x}_t\right) \int_{S^2} \rho(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{v}}) L(\mathbf{x}_t, \overrightarrow{\mathbf{v}})\,\mathrm{d}\overrightarrow{\mathbf{v}} \right) \mathrm{d}t$$

Emissive term ▼

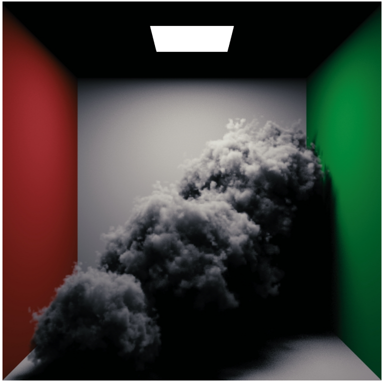
$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \int_a^b e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} L_e\left(\mathbf{x}_t, \overrightarrow{\mathbf{w}}\right) \mathrm{d}t$$

Scattering term ▼

$$L(\mathbf{x}, \overrightarrow{\mathbf{w}}) = \int_a^b \sigma_s\left(\mathbf{x}_t\right) e^{-\int_0^t \sigma_t(\mathbf{x}_s)\,\mathrm{d}s} \int_{S^2} \rho(\overrightarrow{\mathbf{w}}, \overrightarrow{\mathbf{v}}) L(\mathbf{x}_t, \overrightarrow{\mathbf{v}})\,\mathrm{d}\overrightarrow{\mathbf{v}}\,\mathrm{d}t$$

Multiple Scattering

- Use discrete pdf to generate sampling locations for path tracing
- Works well for 1-2 bounces



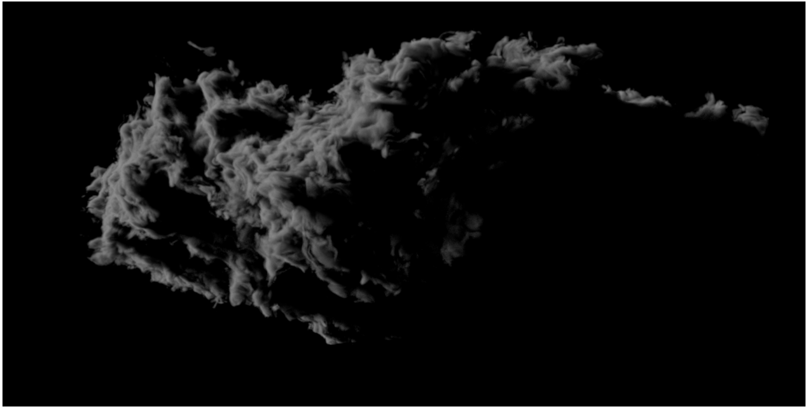
No bounce (2m15s)

1 Bounce (9m03s)

2 Bounces (18m14s)

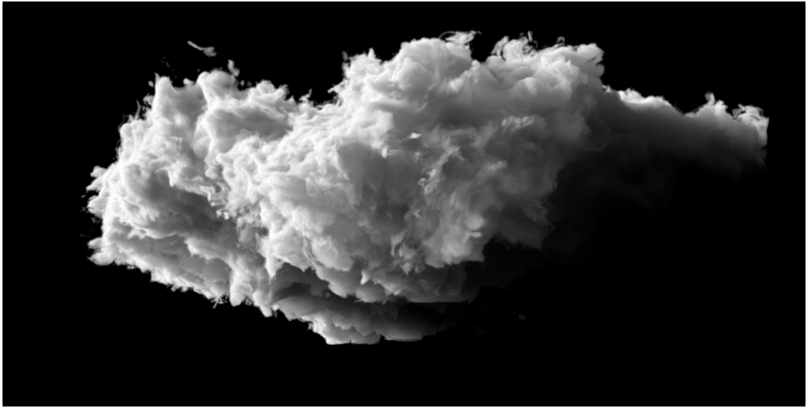
Multiple Scattering Approximation

- High albedo media require many bounces



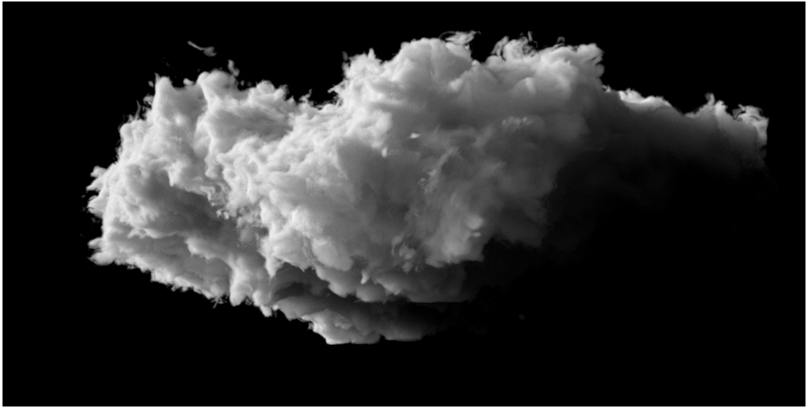
Multiple Scattering Approximation

- High albedo media require many bounces
- Approximate higher order bounces by changing density along shadow rays



Multiple Scattering Approximation

- High albedo media require many bounces
- Approximate higher order bounces by changing density along shadow rays
- Combine with one real bounce to achieve more natural diffusion



Implementation Details

- Ray tracer reports *interval* hits with volume primitives
- Motion blurred media, lights, camera handled automatically
- Only two parameters exposed: step size and light samples (very intuitive for artists)
- Can unify handling of transparent objects into the discrete pdf (hair in volumes)

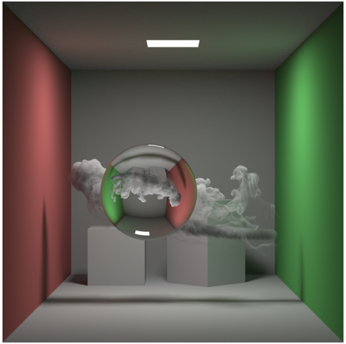
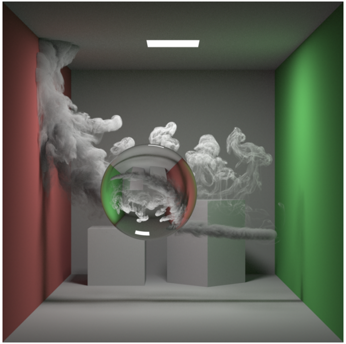
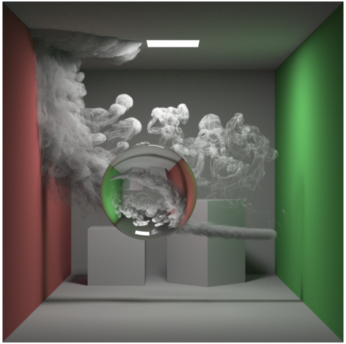
Results

Videos

Future Work

- Generalize to bi-directional methods
- Improve primitive bounds for motion blur
- GPU implementation

Thanks for listening!

Questions?